2017/07/11-12
流砂・河床変動に関する若手勉強会

3次元反砂堆に関する実験とモデル：線形安定解析と数値計算

寒地土木研究所

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2017/03/17
第61回水工学講演会

三次元反砂堆のモデル化：三角状水面波列との関連性

Modeling the three-dimensional river antidunes: Implications for the triangle-shape water surface wave trains

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Background

- A triangle-shape water surface wave trains
  - Steep rivers
  - Supercritical flows

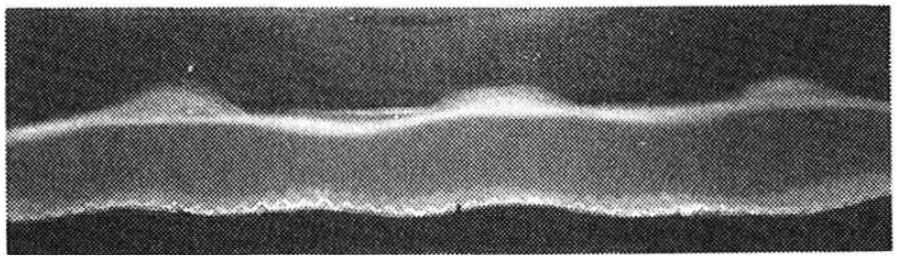
Flood in Toyohira River, Aug, 1981
（石狩川流域誌より）

Flood in the Bebetsu River, Aug, 2013
（Courtesy of Y. Shimizu）

These surface waves may destabilize river embankments, bridge pier, and man-made structures in rivers

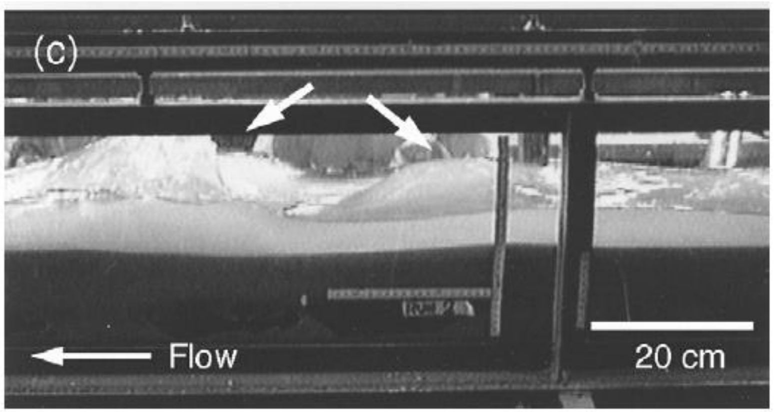
Background

- Previous study
  - Stationary free surface waves associated with irregularly-perturbed bed (Yamada et al., 1984)



Yamada et al., 1984

  - Free surface – antidune resonance (Hasegawa, 1997)

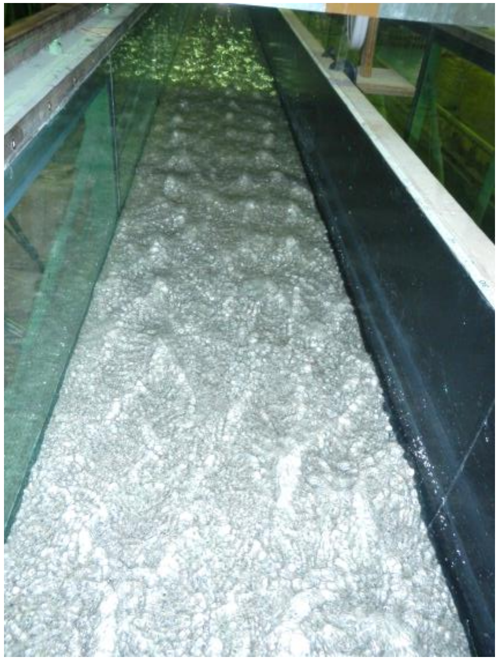


Yokokawa et al., 2010

⇒ Bed perturbation (i.e., antidunes) has an important role in the formation of surface waves.

3D antidunes

- Theoretical study (linear theory)
  - 2D antidunes: e.g., Kennedy, 1961, 1963, 1969; Reynolds,1965; Hayashi,1970; Parker, 1975; Colombini, 2004; Izumi and Parker, 2009…..
  - 3D antidunes: e.g., Kuroki et al., 1985; Kanbayashi and Hasegawa, 1996; Colombini and Stocchino, 2012.
- Numerical simulations
  - 2D antidunes: Onda and Hosoda, 2004; Uchida and Fukuoka, 2013; Yamaguchi et al., 2015; Bohorquez and Ancey, 2015; Olsen, 2016…
  - 3D antidunes: …



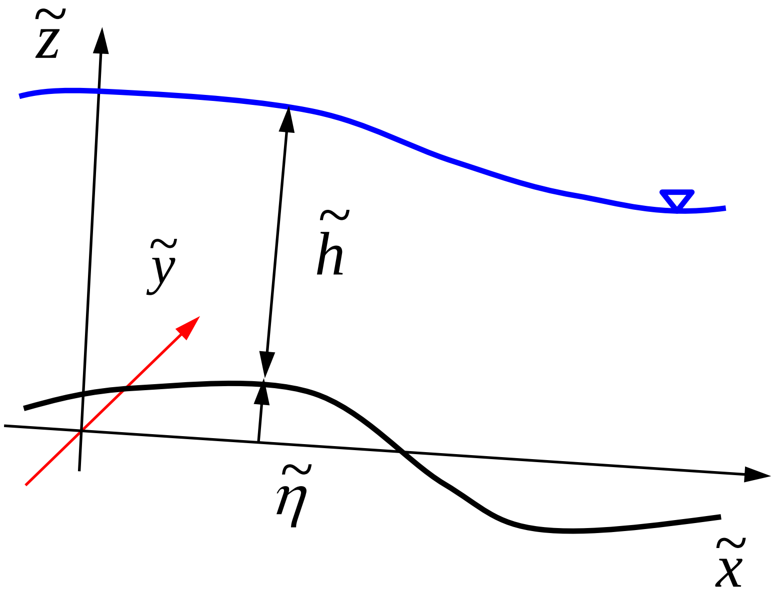
Inoue et al., 2015

- Present study
  - We perform a linear stability analysis of 3D antidunes, and compare the theoretical result (i.e., streamwise, spanwise wavenumbers) with the experimental results.
  - Some implications for the triangle-shape free water surface wave trains.

Model

- Hydrodynamic model
  - A Boussinesq model (Onda and Hosoda, 2004)
  - A Phase difference between flow velocity and shear stress

$$\tilde{\tau} = \tilde{\rho} C_f \tilde{V}_b^{\,2} \left(1 - \Gamma \frac{\partial \tilde{h}}{\partial \tilde{s}} + \Delta \frac{\partial \tilde{\eta}}{\partial \tilde{s}} \right)$$



- Morphodynamic model
  - Non-equilibrium bedload transport model

(Uchida and Fukuoka, 2013)

$$\frac{\partial \tilde{q}_{bx}}{\partial \tilde{t}} + \frac{\partial \tilde{u}_{px} \tilde{q}_{bx}}{\partial \tilde{x}} + \frac{\partial \tilde{u}_{py} \tilde{q}_{bx}}{\partial \tilde{y}} = \frac{1}{\tilde{l}_s} \left(\tilde{u}_{pxe} \tilde{q}_{be} - \tilde{u}_{px} \tilde{q}_b \right)$$

A Boussinesq equation

$$\frac{\partial \tilde{U}\tilde{h}}{\partial \tilde{x}} + \frac{\partial \tilde{V}\tilde{h}}{\partial \tilde{y}} = 0$$

$$\tilde{U}\frac{\partial \tilde{U}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{U}}{\partial \tilde{y}} = \tilde{g}\sin\theta - \frac{1}{\tilde{h}}\frac{\partial}{\partial \tilde{x}}\int_{\tilde{\eta}}^{\tilde{H}} \frac{\tilde{p}}{\tilde{\rho}}\,d\tilde{z} - \frac{1}{\tilde{h}}\left.\frac{\tilde{p}}{\tilde{\rho}}\right|_{\tilde{z}=\tilde{\eta}} \frac{\partial \tilde{\eta}}{\partial \tilde{x}} - \frac{\tilde{\tau}_x}{\tilde{\rho}\tilde{h}}$$

$$\tilde{U}\frac{\partial \tilde{V}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{V}}{\partial \tilde{y}} = -\frac{1}{\tilde{h}}\frac{\partial}{\partial \tilde{y}}\int_{\tilde{\eta}}^{\tilde{H}} \frac{\tilde{p}}{\tilde{\rho}}\,d\tilde{z} - \frac{1}{\tilde{h}}\left.\frac{\tilde{p}}{\tilde{\rho}}\right|_{\tilde{z}=\tilde{\eta}} \frac{\partial \tilde{\eta}}{\partial \tilde{y}} - \frac{\tilde{\tau}_y}{\tilde{\rho}\tilde{h}}$$

$$\frac{\tilde{p}}{\tilde{\rho}} = \tilde{g}\tilde{h}\cos\theta(1-\zeta) - \left(\tilde{B}^2 + 2\tilde{A}\tilde{B} + \zeta^2\tilde{A}^2\right)$$

$$+(1-\zeta)\tilde{U}\tilde{h}\frac{\partial \tilde{B}}{\partial \tilde{x}} + \frac{1}{2}\left(1-\zeta^2\right)\tilde{U}\tilde{h}\frac{\partial \tilde{A}}{\partial \tilde{x}}$$

$$+(1-\zeta)\tilde{V}\tilde{h}\frac{\partial \tilde{B}}{\partial \tilde{y}} + \frac{1}{2}\left(1-\zeta^2\right)\tilde{V}\tilde{h}\frac{\partial \tilde{A}}{\partial \tilde{y}}$$

$$\tilde{A} = \tilde{U}\frac{\partial \tilde{h}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{h}}{\partial \tilde{y}}, \qquad \tilde{B} = \tilde{U}\frac{\partial \tilde{\eta}}{\partial \tilde{x}} + \tilde{V}\frac{\partial \tilde{\eta}}{\partial \tilde{y}}, \qquad \zeta = \frac{\tilde{z} - \tilde{h}}{\tilde{h}}$$

Sediment transport model

- Exner Eq.

$$(1-\lambda)\frac{\partial\tilde{\eta}}{\partial\tilde{t}}+\frac{\partial\tilde{q}_{bx}}{\partial\tilde{x}}+\frac{\partial\tilde{q}_{by}}{\partial\tilde{y}}=0$$

- Non-equilibrium bedload transport model

$$\frac{\partial\tilde{q}_{bx}}{\partial\tilde{t}}+\frac{\partial\tilde{u}_{px}\tilde{q}_{bx}}{\partial\tilde{x}}+\frac{\partial\tilde{u}_{py}\tilde{q}_{bx}}{\partial\tilde{y}}=\frac{1}{\tilde{l}_s}\left(\tilde{u}_{pxe}\tilde{q}_{be}-\tilde{u}_{px}\tilde{q}_b\right)$$

$\tilde{l}_s$: step length

$$\frac{\partial\tilde{q}_{by}}{\partial\tilde{t}}+\frac{\partial\tilde{u}_{px}\tilde{q}_{by}}{\partial\tilde{x}}+\frac{\partial\tilde{u}_{py}\tilde{q}_{by}}{\partial\tilde{y}}=\frac{1}{\tilde{l}_s}\left(\tilde{u}_{pye}\tilde{q}_{be}-\tilde{u}_{py}\tilde{q}_b\right)$$

$$\tilde{q}_{be}=4\left(\tau_*-\tau_{*c}\right)^{3/2}\sqrt{s\tilde{g}\tilde{d}^3}$$

$$\left(\tilde{u}_{px},\tilde{u}_{py},\tilde{u}_{pxe},\tilde{u}_{pye}\right)=\frac{1}{\tilde{L}_a(1-\lambda)}\left(\tilde{q}_{bx},\tilde{q}_{by},\tilde{q}_{bxe},\tilde{q}_{bye}\right)$$

$\tilde{L}_a$: Bedload layer thickness

$$\tilde{q}_{bex}=\frac{1}{\sqrt{\tilde{U}^2+\tilde{V}^2}}\left(\tilde{U}\tilde{q}_{be}-\tilde{V}\tilde{q}_{ben}\right),\quad \tilde{q}_{bey}=\frac{1}{\sqrt{\tilde{U}^2+\tilde{V}^2}}\left(\tilde{V}\tilde{q}_{be}+\tilde{U}\tilde{q}_{ben}\right)$$

$$\tilde{q}_{ben}=\tilde{q}_{be}\left(\frac{\tilde{U}_{bn}}{\tilde{V}_b}-\sqrt{\frac{\tau_{*c}}{\mu_s\mu_k\tau_*}}\frac{\partial\tilde{\eta}}{\partial\tilde{n}}\right)$$

Nondimensionalization

$$\left(\tilde{x},\tilde{y},\tilde{h},\tilde{\eta}\right)=\tilde{h}_0\left(x,y,h,\eta\right)$$

$$\left(\tilde{U},\tilde{V}\right)=\tilde{U}_0\left(U,V\right)$$

$$\left(\tilde{q}_{be},\tilde{q}_{bx},\tilde{q}_{by}\right)=\tilde{q}_{b0}\left(q_{be},q_{bx},q_{by}\right)$$

$$\tilde{q}_{b0}=4\left(\tau_{*0}-\tau_{*c}\right)^{3/2}\sqrt{sg\tilde{d}^{3}}$$

$$\tau_{*0}=\frac{C_f U_0^{\ 2}}{sg\tilde{d}}$$

$$\tilde{T}=\frac{\left(1-\lambda\right)\tilde{h}_0^{\ 2}}{\tilde{q}_{b0}}$$

$()_0$: value at the condition of uniform flow

Linearization

$$\left(U,V,h,\eta,q_{bx},q_{by}\right)=\left(1,0,1,0,1,0\right)+\varepsilon\left(U_1,V_1,h_1,\eta_1,q_{bx1},q_{by1}\right)$$

Linearized governing equations

$$\frac{\partial U_1}{\partial x}+\frac{\partial h_1}{\partial x}+\frac{\partial V_1}{\partial y}=0$$

$$\frac{\partial U_1}{\partial x}=-\frac{1}{F_r^{\,2}}\left(\frac{\partial h_1}{\partial x}+\frac{\partial \eta_1}{\partial x}\right)-C_f\left(2U_1-h_1\right)-\frac{1}{2}\frac{\partial^3\eta_1}{\partial x^3}-\frac{1}{3}\frac{\partial^3 h_1}{\partial x^3}$$

$$\frac{\partial V_1}{\partial x}=-\frac{1}{F_r^{\,2}}\left(\frac{\partial h_1}{\partial y}+\frac{\partial \eta_1}{\partial y}\right)-C_f V_1-\frac{1}{2}\frac{\partial^3\eta_1}{\partial x^2\partial y}-\frac{1}{3}\frac{\partial^3 h_1}{\partial x^2\partial y}$$

$$\frac{\partial \eta_1}{\partial t}+\frac{\partial q_{bx1}}{\partial x}+\frac{\partial q_{by1}}{\partial y}=0$$

$$2\frac{\partial q_{bx1}}{\partial x}+\frac{\partial q_{by1}}{\partial y}=\delta\left(6\Theta U_1-3\Theta\Gamma\frac{\partial h_1}{\partial x}+3\Theta\Delta\frac{\partial \eta_1}{\partial x}-3\Theta\frac{\partial^2 h_1}{\partial x^2}-\Theta\frac{\partial^2\eta_1}{\partial x^2}-2q_{bx1}\right)$$

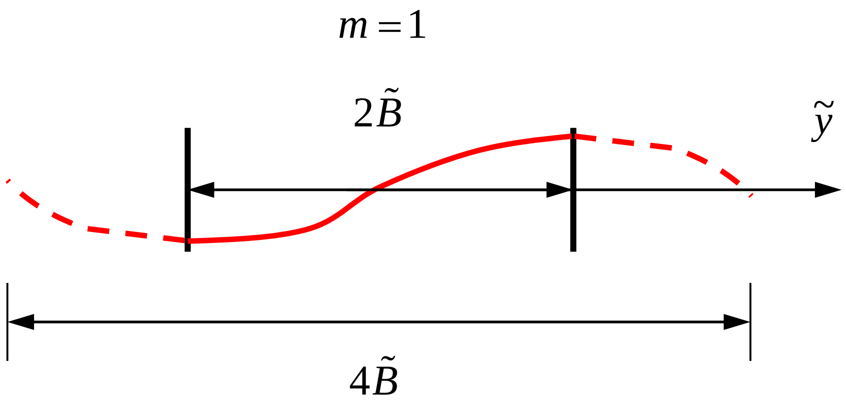
$$\frac{\partial q_{by1}}{\partial x}=\delta\left(V_1-\frac{1}{\mu_c}\sqrt{\frac{\tau_{*c}}{\tau_{*0}}}\frac{\partial \eta_1}{\partial y}-q_{by1}\right)$$

$$F_r=\frac{\tilde{U}_0}{\sqrt{g\tilde{h}_0}},\quad \Theta=\frac{\tau_{*0}}{\tau_{*0}-\tau_{*c}},\quad \delta=\frac{\tilde{h}_0}{\tilde{l}_s}$$

3D bed perturbation for linear stability analysis

$$k_x = \frac{2\pi \tilde{h}_0}{\tilde{L}_x}$$

$$k_y = \frac{2\pi \tilde{h}_0}{\tilde{L}_y} = \frac{2\pi \tilde{h}_0}{\frac{4}{m}\tilde{B}} = \frac{m\pi}{2\beta}$$



$$\left(u_1, h_1, \eta_1, q_{bx1}\right) = \left(u_1, h_1, \eta_1, q_{bx1}\right) e^{\Omega t} e^{i\left(k_x x - \omega t\right)} \sin\left(k_y y\right) + c.c. \qquad m\text{: odd}$$

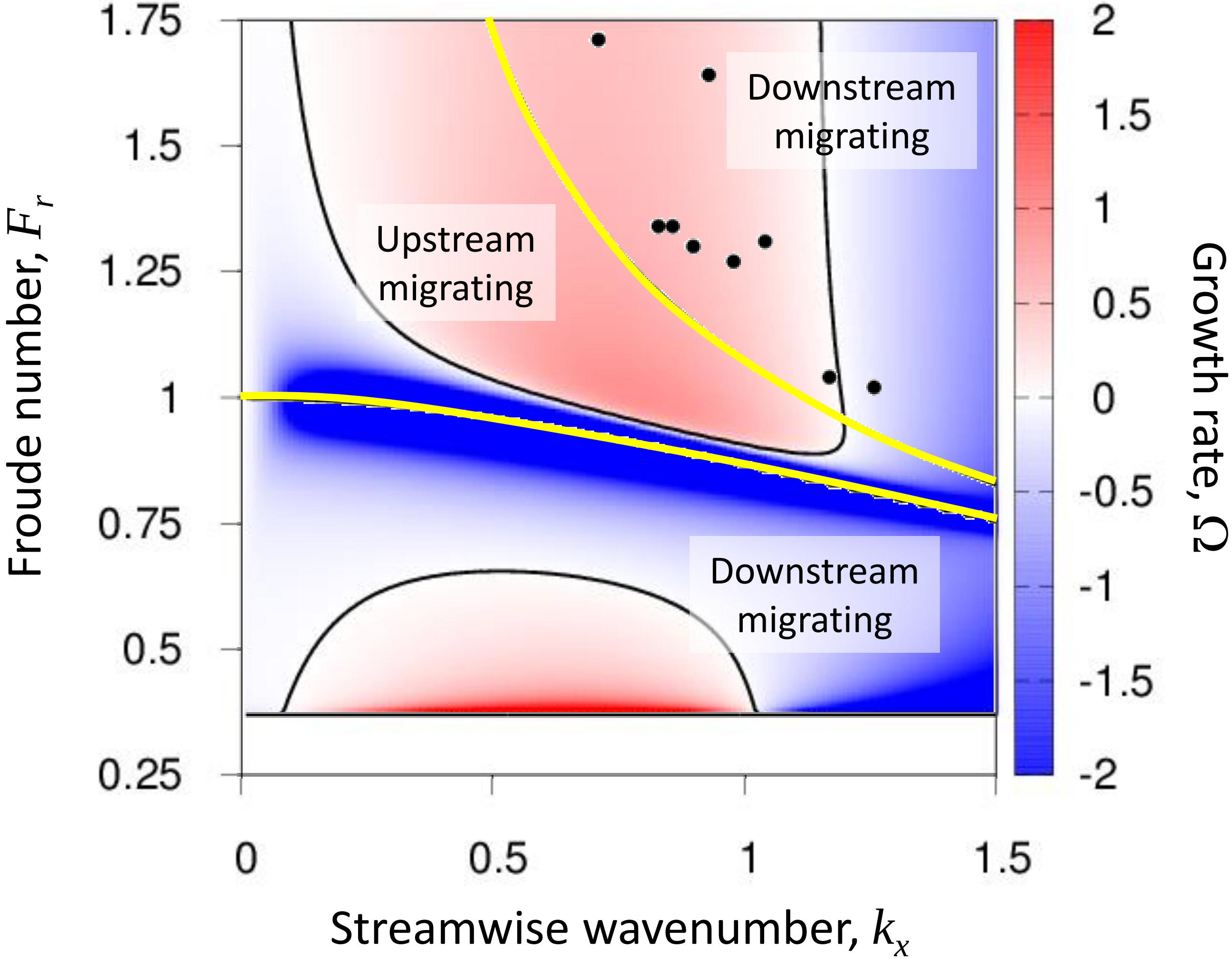
$$= \left(u_1, h_1, \eta_1, q_{bx1}\right) e^{\Omega t} e^{i\left(k_x x - \omega t\right)} \cos\left(k_y y\right) + c.c. \qquad m\text{: even}$$

$$\left(v_1, q_{by1}\right) = \left(v_1, q_{by1}\right) e^{\Omega t} e^{i\left(k_x x - \omega t\right)} \cos\left(k_y y\right) + c.c. \qquad m\text{: odd}$$

$$= \left(v_1, q_{by1}\right) e^{\Omega t} e^{i\left(k_x x - \omega t\right)} \sin\left(k_y y\right) + c.c. \qquad m\text{: even}$$

Essentially, in a same way to free bar instability analysis.

2D dune/anditune instability (k_y=0)



$C_f = 15^{-2}$

$l_s = 100$

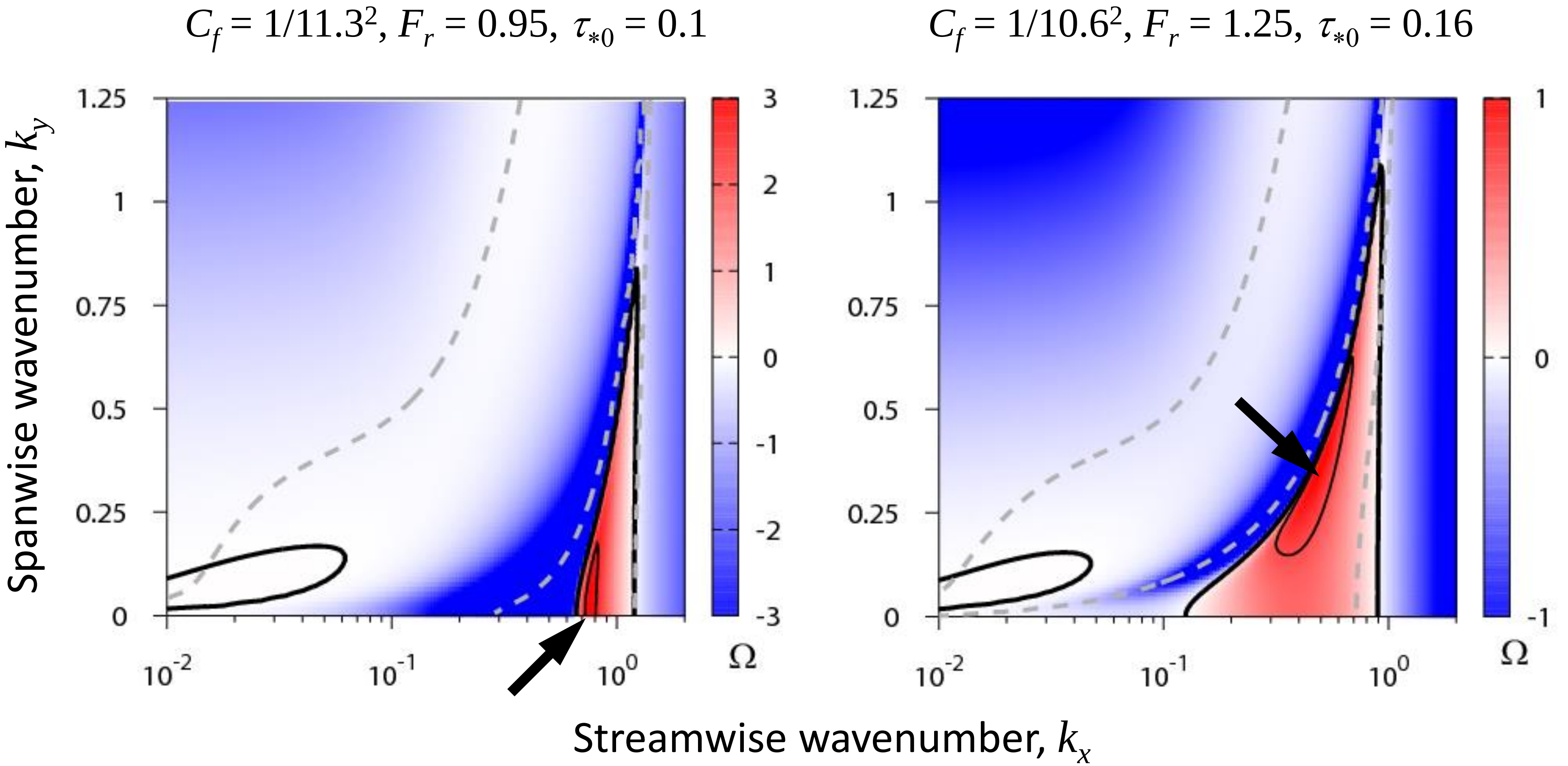
$\Gamma = 1.5$

$\Delta = 1.5$

● Inoue et al., 2015

The present model gives a similar instability diagram obtained by a shear flow model.

3D antidune instability



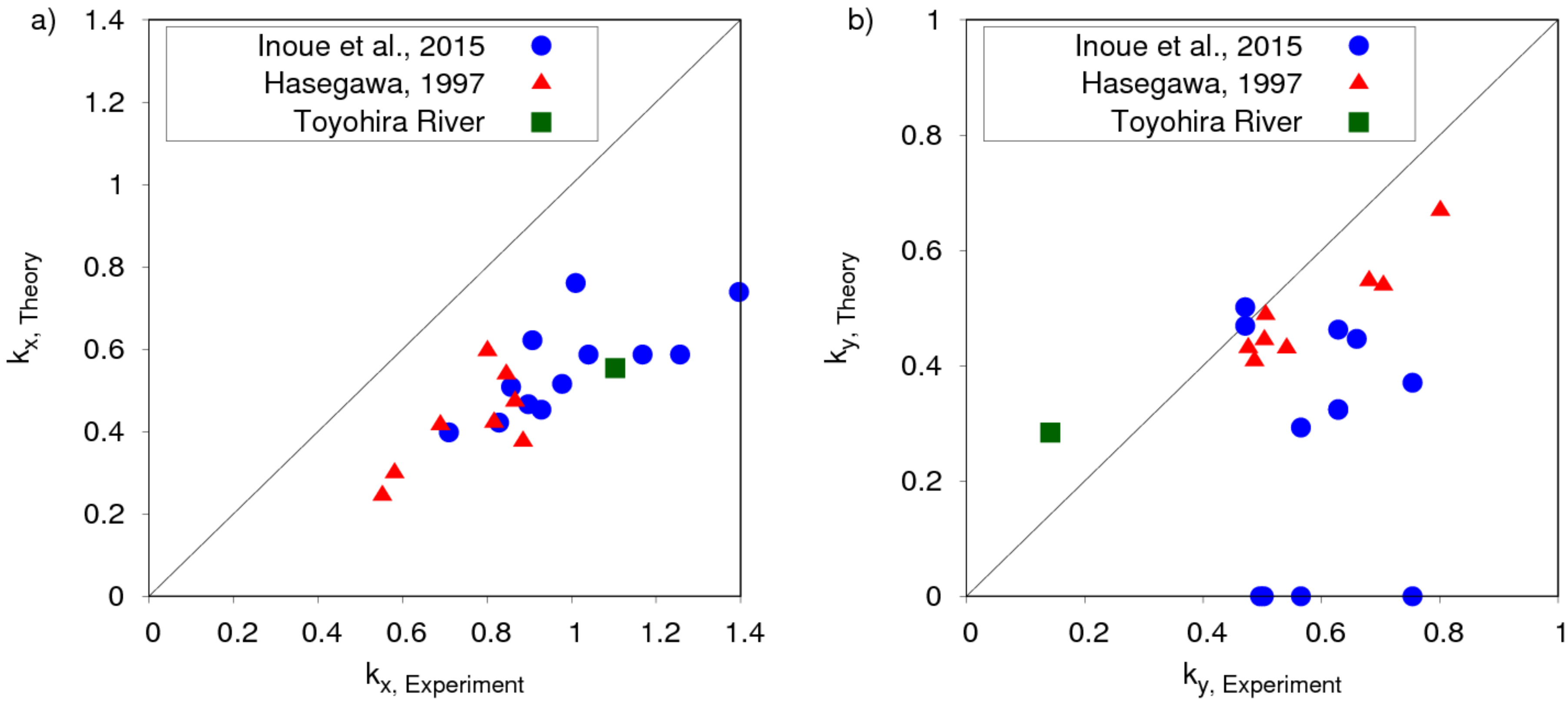
k_y is zero at Ω_{max}

k_y is NOT zero at Ω_{max}

2D antidunes

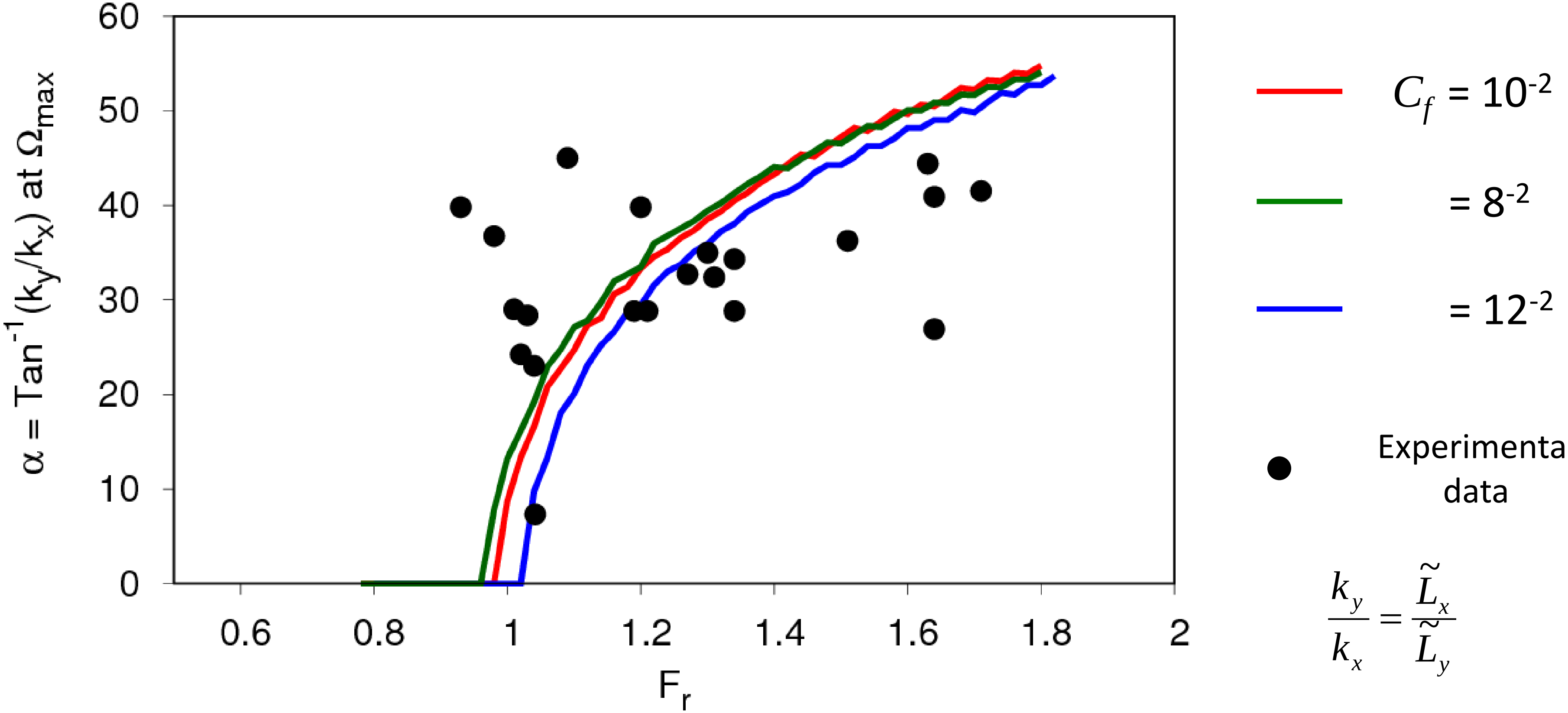
3D antidunes

Comparison with the experimental data: Implications for the triangle-shape wave trains

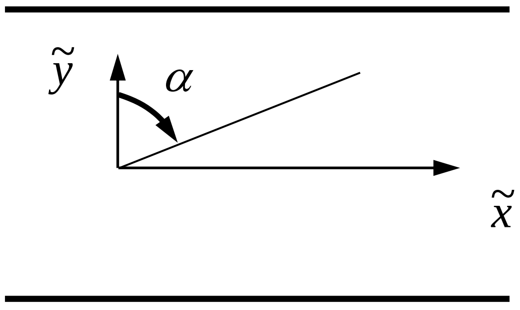


- The model underpredicts the observed wavenumbers of antidunes (The model overpredict the wavelength of antidunes).
- The hydrodynamic model used here (i.e., Boussinesq model) is not very good for analysis of short-scale waves.

Dominant instability mode: 2D/3D



$$\frac{k_y}{k_x} = \frac{\tilde{L}_x}{\tilde{L}_y}$$



- Dominant antidune mode shifts from 2D to 3D with increasing the Froude number.
- The discrepancy at low Fr:
  - Short antidune at low Fr
  - Nonlinear interactions

数値計算への展開

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ブシネスクモデル

$$\frac{\partial h}{\partial t}+\frac{\partial Uh}{\partial x}+\frac{\partial Vh}{\partial y}=0$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=g\sin\theta-g\left(\frac{\partial h}{\partial x}+\frac{\partial \eta}{\partial x}\right)-\frac{1}{h}\frac{\partial}{\partial x}\int_{\eta}^{\mathrm{H}}\frac{p}{\rho}dz-\frac{\tau_x}{\rho h}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-g\left(\frac{\partial h}{\partial y}+\frac{\partial \eta}{\partial y}\right)-\frac{1}{h}\frac{\partial}{\partial y}\int_{\eta}^{\mathrm{H}}\frac{p}{\rho}dz-\frac{\tau_y}{\rho h}$$

$$\int_{\eta}^{\mathrm{H}}\frac{p}{\rho}dz=-h^3\left(B+\frac{1}{3}\right)\left(\frac{\partial}{\partial x}\frac{\partial U}{\partial t}+\frac{\partial}{\partial y}\frac{\partial V}{\partial t}\right)-\frac{h^3}{3}\left[U\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 V}{\partial x\partial y}\right)+V\left(\frac{\partial^2 U}{\partial x\partial y}+\frac{\partial^2 V}{\partial y^2}\right)\right]$$
$$+\frac{h^2}{2}\left(U^2\frac{\partial^2\eta}{\partial x^2}+2UV\frac{\partial^2\eta}{\partial x\partial y}+V^2\frac{\partial^2\eta}{\partial y^2}\right)-\frac{h^3}{3}Bg\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)$$

B: Madsenらによる補正項
（ここでは，1/21とした）

分散項は三回微分を含むため離散化や収束計算が難しい．．．

ブシネスク式の数値計算

鴨原・藤間，2007を参考に．．

$$\frac{\partial U}{\partial t} + f_x = \frac{1}{h}\frac{\partial \varphi}{\partial x}$$

$$\frac{\partial V}{\partial t} + f_y = \frac{1}{h}\frac{\partial \varphi}{\partial y}$$

f_x, f_y : 分散項以外の項
（移流項，静水圧項，河床勾配項，摩擦項）

$$\varphi = \int_{\eta}^{H} \frac{p}{\rho} dz = h^3\left(B + \frac{1}{3}\right)\left(\frac{\partial}{\partial x}\frac{\partial U}{\partial t} + \frac{\partial}{\partial y}\frac{\partial V}{\partial t}\right) + \frac{h^3}{3}\left[U\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 V}{\partial x \partial y}\right) + V\left(\frac{\partial^2 U}{\partial x \partial y} + \frac{\partial^2 V}{\partial y^2}\right)\right]$$

$$- \frac{h^2}{2}\left(U^2\frac{\partial^2 \eta}{\partial x^2} + 2UV\frac{\partial^2 \eta}{\partial x \partial y} + V^2\frac{\partial^2 \eta}{\partial y^2}\right) + \frac{h^3}{3}Bg\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right)$$

$$= h^3\left(B + \frac{1}{3}\right)\left[\frac{\partial}{\partial x}\left(\frac{1}{h}\frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{h}\frac{\partial \varphi}{\partial y}\right) - \left(\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y}\right)\right] + \frac{h^3}{3}\left[U\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 V}{\partial x \partial y}\right) + V\left(\frac{\partial^2 U}{\partial x \partial y} + \frac{\partial^2 V}{\partial y^2}\right)\right]$$

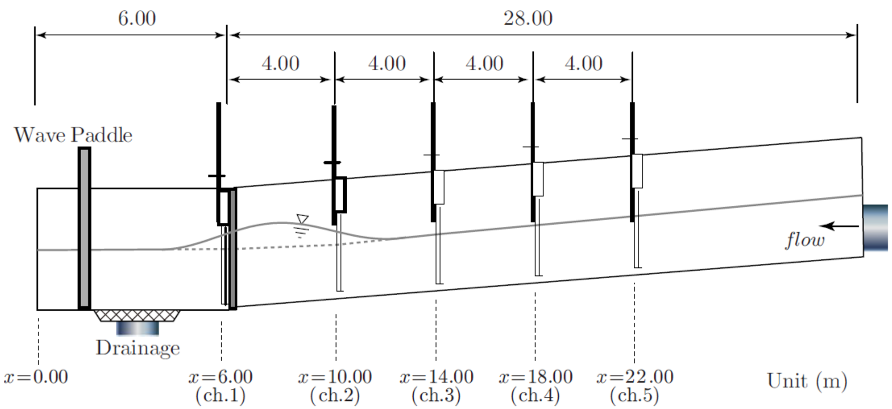
$$- \frac{h^2}{2}\left(U^2\frac{\partial^2 \eta}{\partial x^2} + 2UV\frac{\partial^2 \eta}{\partial x \partial y} + V^2\frac{\partial^2 \eta}{\partial y^2}\right) + \frac{h^3}{3}Bg\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right)$$

φ　のポアソン方程式より収束計算を行い，分散項を計算する．
→非静水圧分の圧力場を求めている．

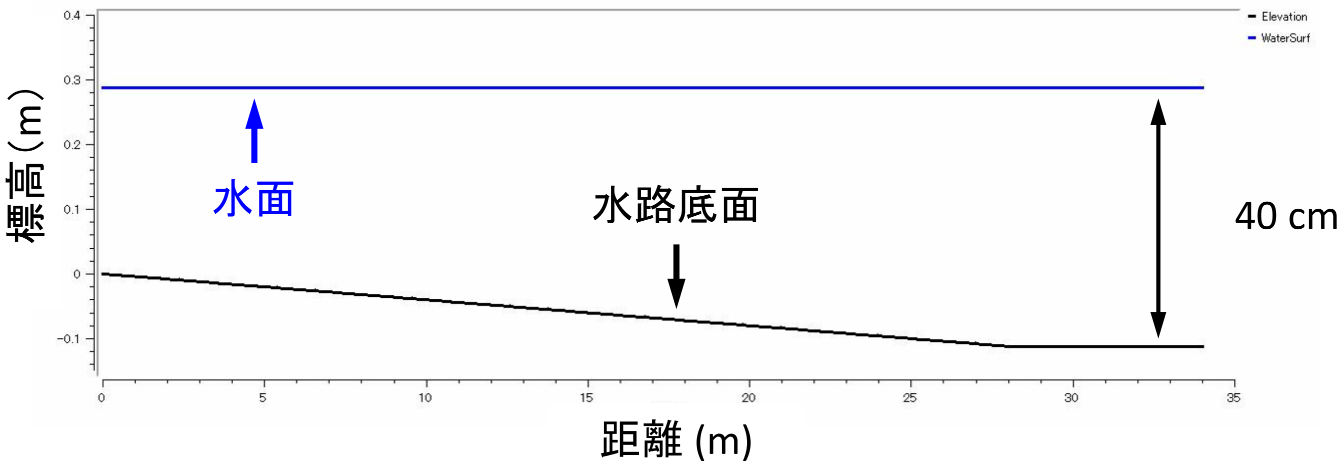
⇒収束方法が明確で，三回微分を離散化しなくてすむ．

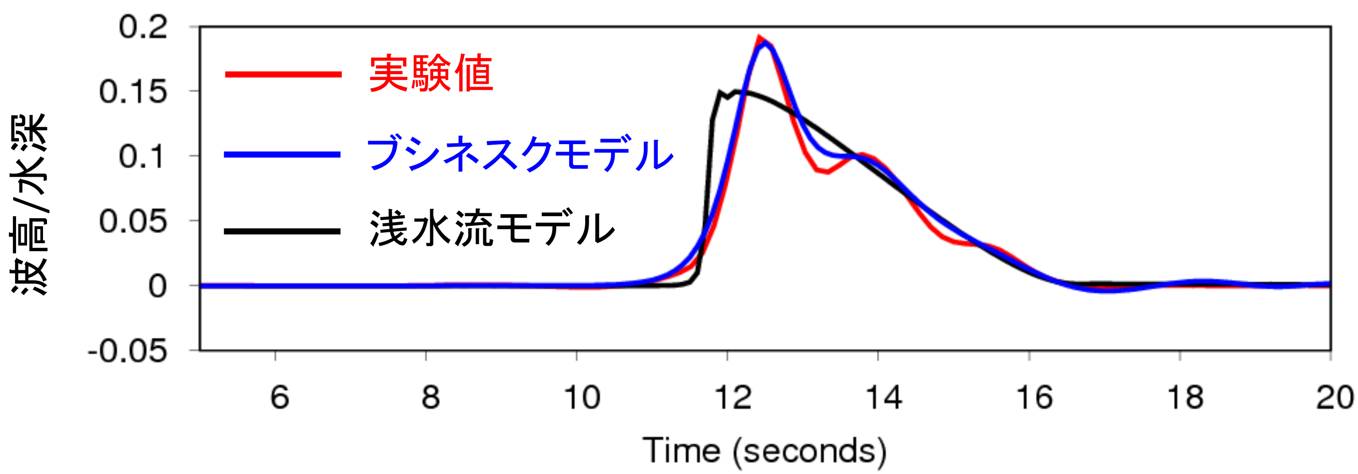
移動床計算の前に．．．水面波の計算

静水面を伝搬する波の計算

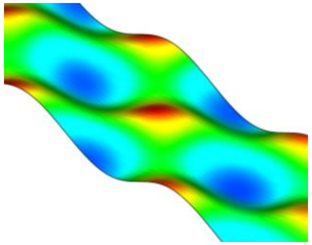


Yasuda, 2009





移動床計算の前に．．．

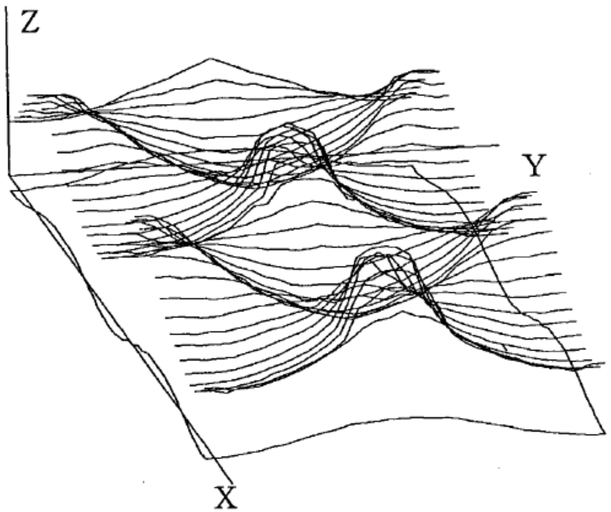


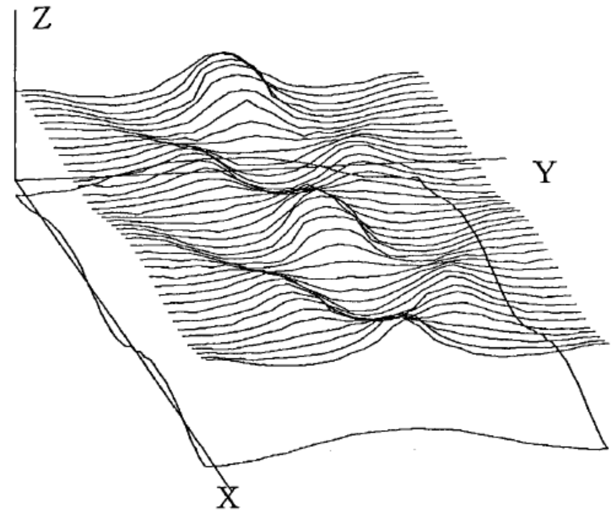
固定床，三次元反砂堆上の水面波 ＝ 三角波

水面波ー河床波　共鳴状態

水面波ー河床波　非共鳴状態

水面形の鳥瞰図





水路中心線上の水面形と流速分布

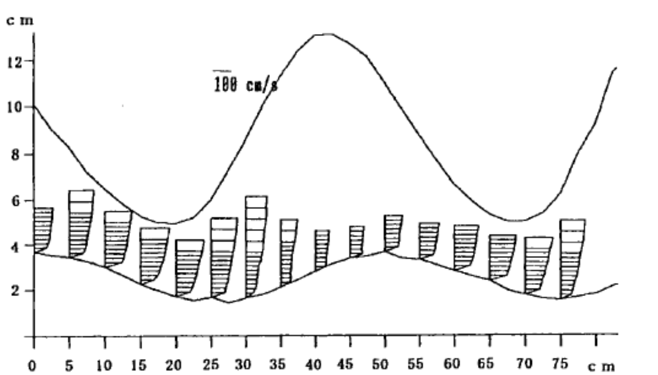


図-20,21. Run-B2における水面波と流速分布

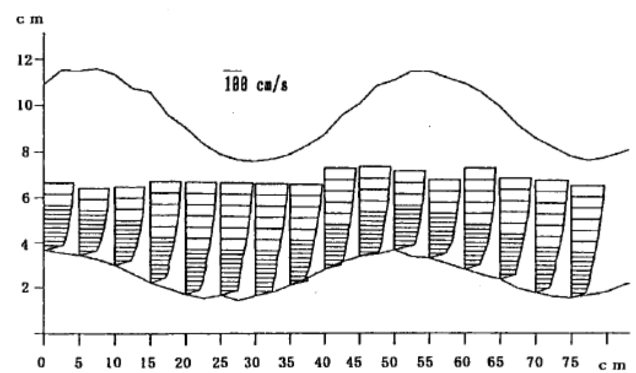
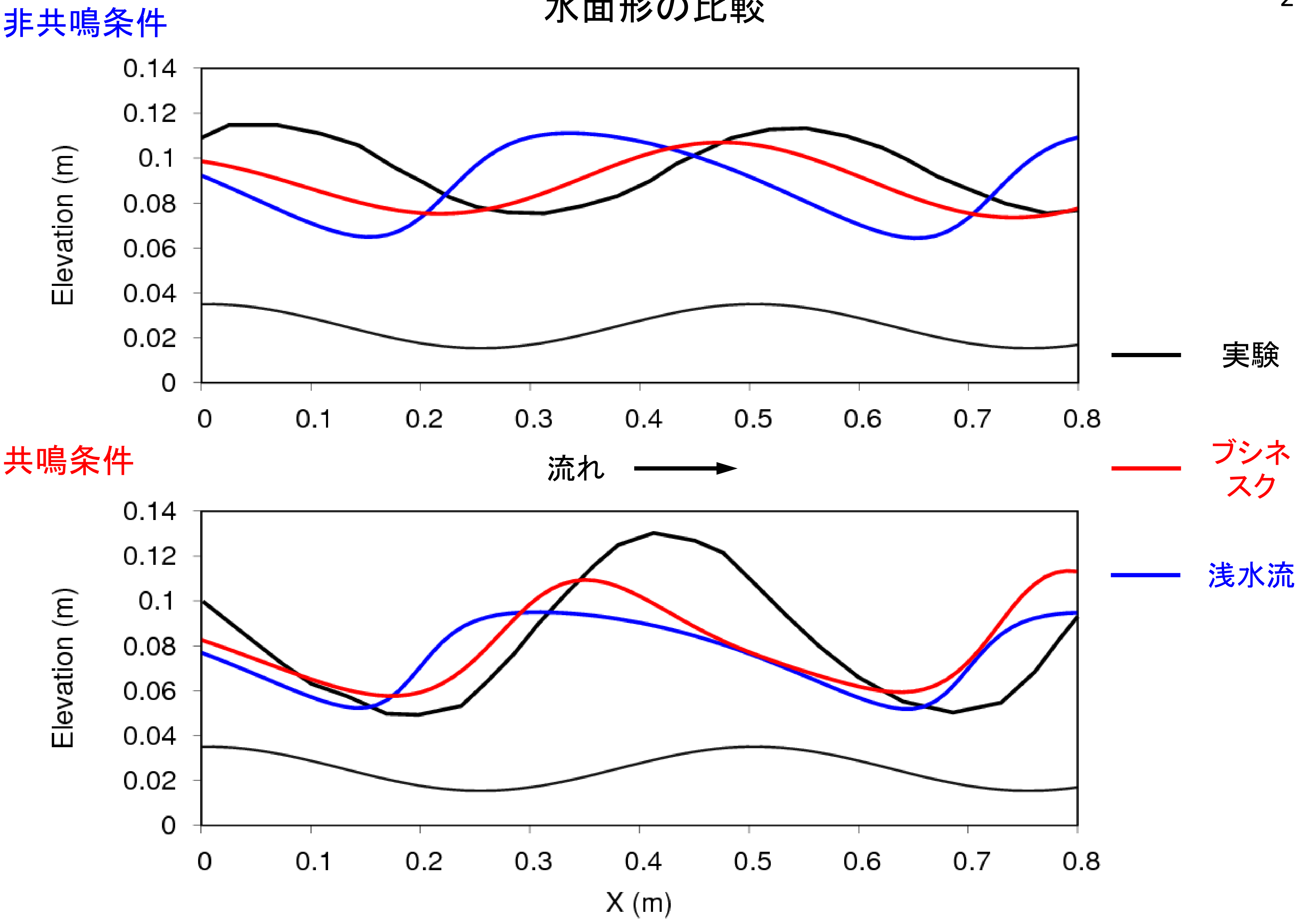


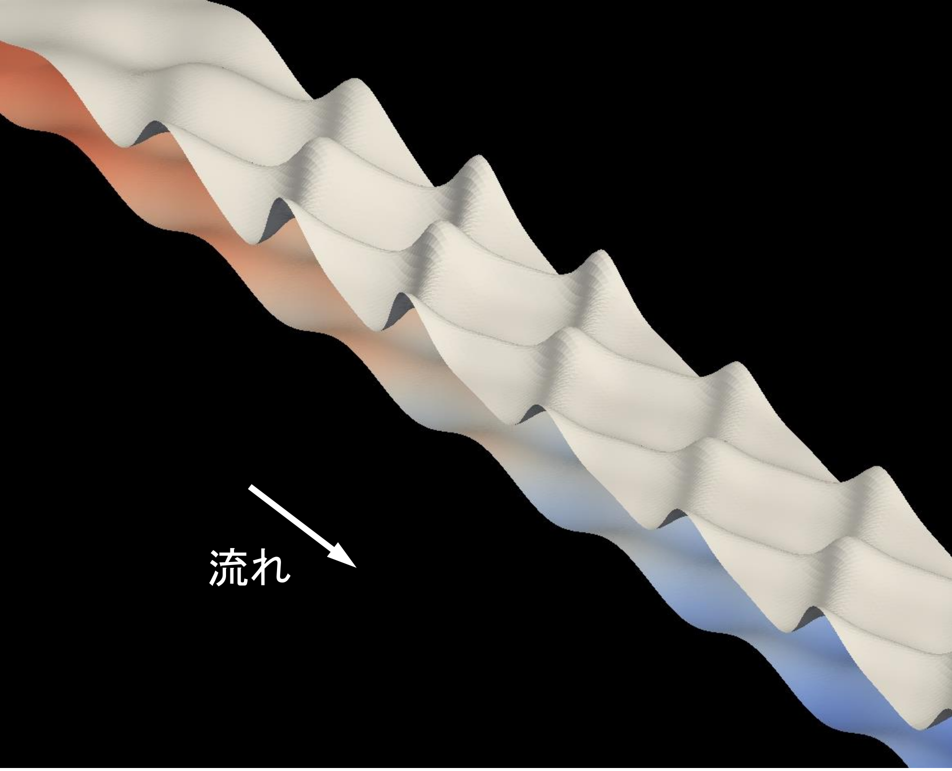
図-18,19. Run-B1における水面波と流速分布

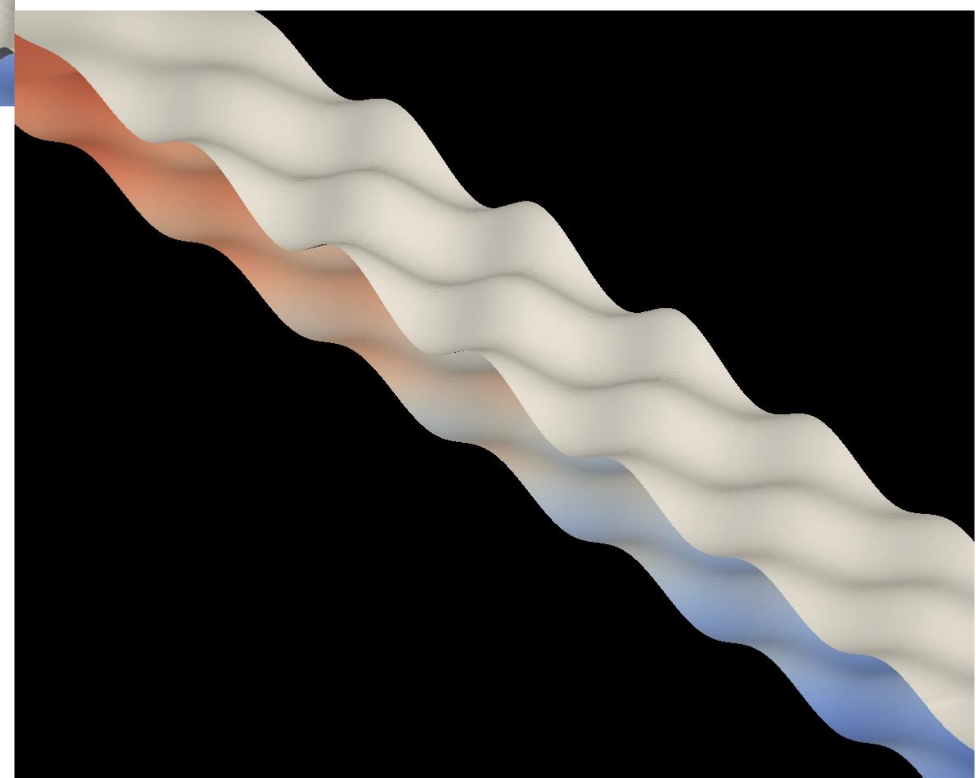
長谷川，1994

水面形の比較


非共鳴条件
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Elevation (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
共鳴条件
流れ
実験
ブシネスク
浅水流
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Elevation (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
X (m)

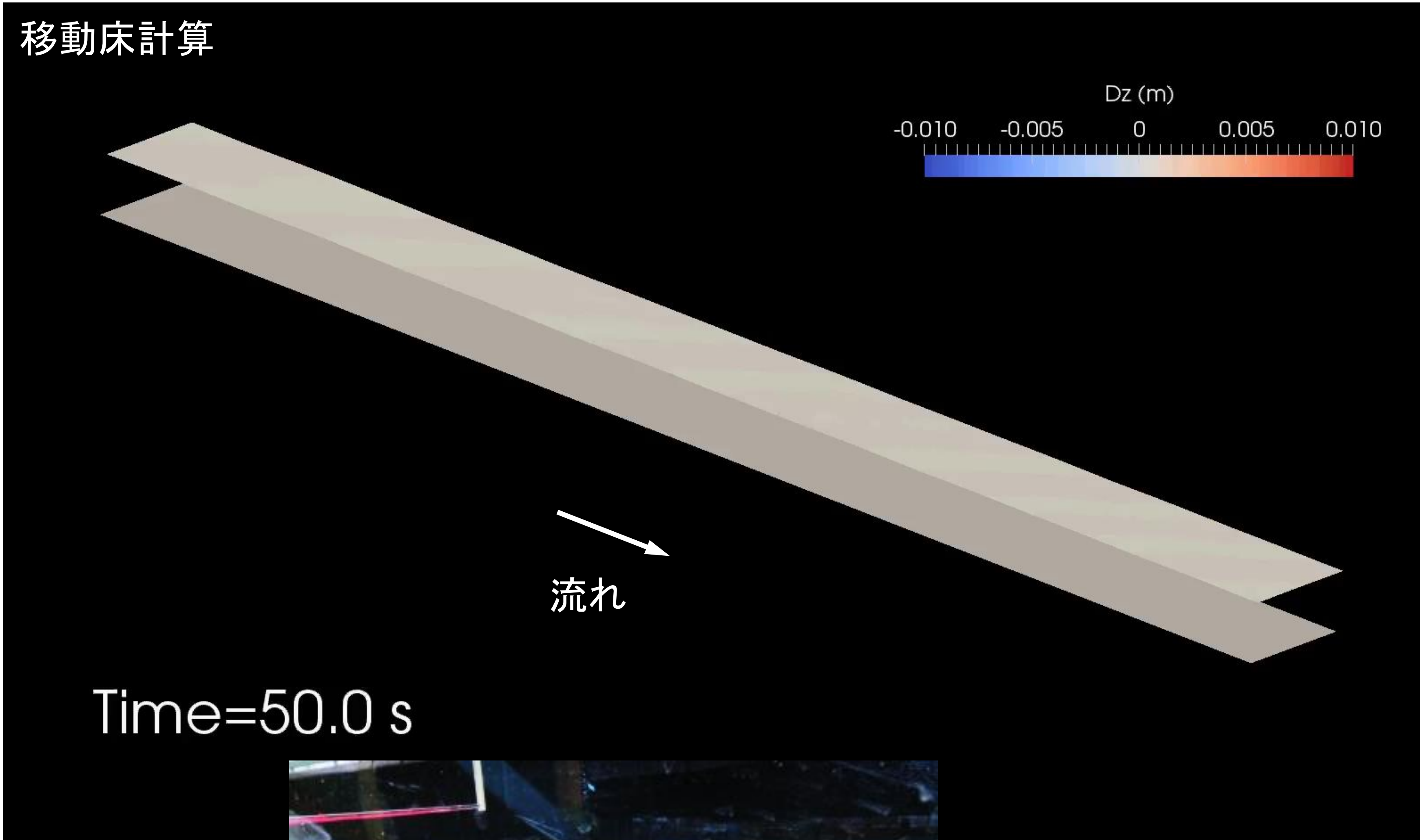
←水面波ー河床波　共鳴状態





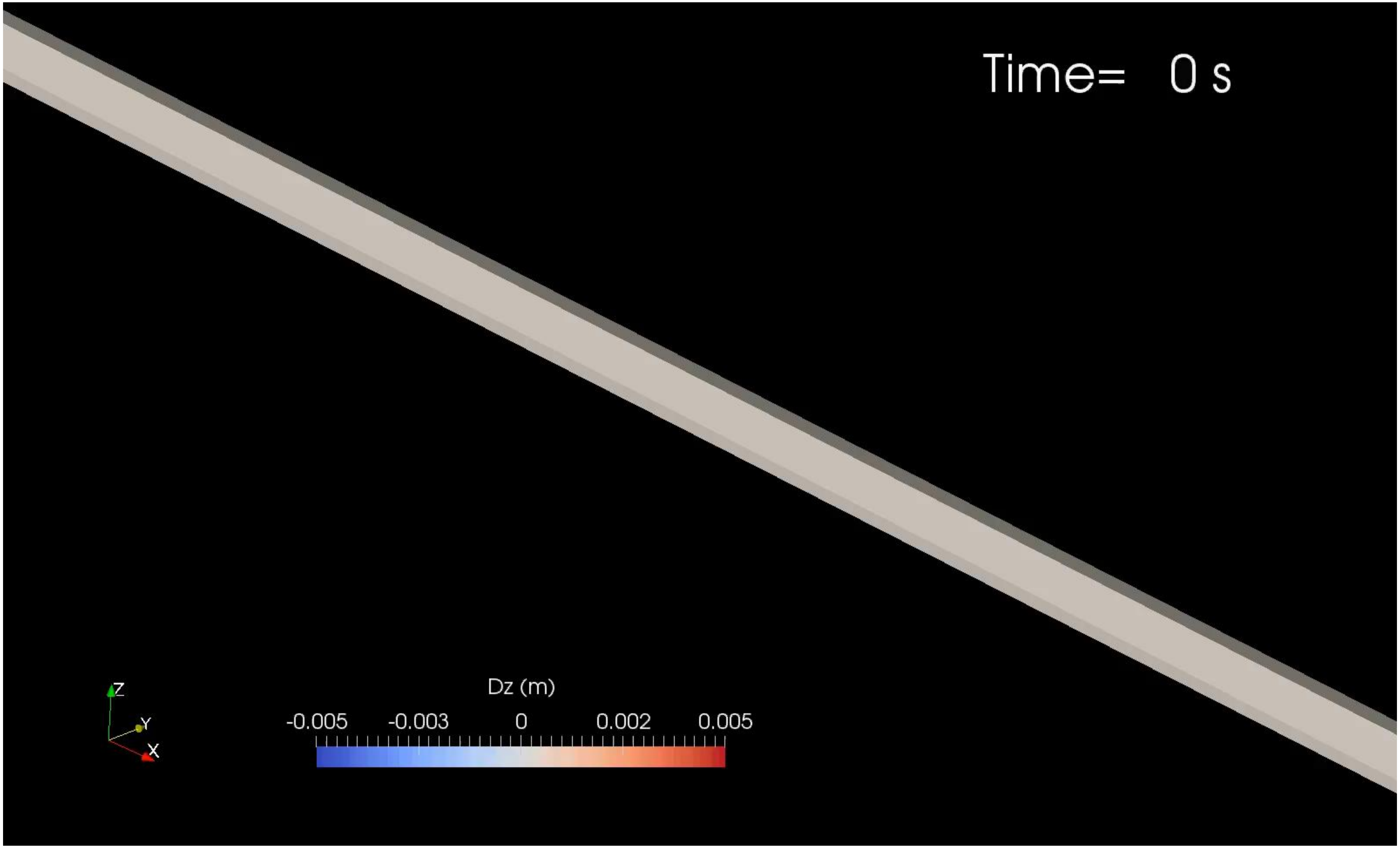
水面波ー河床波　非共鳴状態→

移動床実験に見られる三角波



流量 ＝ 36.8 l/s
河床勾配 ＝ 0.0145
粒径 ＝ 1.42 mm
水路幅 ＝ 0.5 m

移動床計算　川幅/水深が大きい場合の例



数値計算結果まとめ

○良い点

- ✓ 有限波長の反砂堆と水面波
- ✓ 異なる横断方向の列を持った河床波の形成
- ✓ 反砂堆と砂州の共存
- ✓ 発達と消滅の繰り返しの表現

△いまいちな点

- ✓ 射流なのに水面波が単独で上流に伝搬しているように見える（砕波？）
- ✓ 横断方向の列数が計算を続けると減っていく傾向
- ✓ 狙った現象と乖離がある
（流下反砂堆が計算できない）

まとめ

- ブシネスク方程式と非平衡掃流砂モデルから構成される河床変動モデルに対する検討

- 線形安定解析

  → 三次元反砂堆の発生をある程度説明可能

  → 卓越波長は実験と比べ長い傾向

- 数値計算

  → 三次元反砂堆と水面波の発生，砂州との共存など表現可能

  → 横断方向波数の減少が著しい，移動方向は説明できない